

## APPLICATION OF FUZZY TIME SERIES FOR PREDICTING REAL ESTATE PRICES IN TAIWAN

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### Abstract

Real estate has always been the basic demand of settlement and the optimal hedge option of investment and financial management for humanity. It is also the leading industry that drives national economic development while exerting enormous influence on the industrial economic development of the country. Consequently, being able to predict the fluctuation in real estate prices is one of the most important topics for government policy making, industrial investment, and individual commerce. This study established a forecast model of real estate prices by using the analytic method of fuzzy time series. The data source was the time series analytic statistics of the sale contract average unit price registered on the real estate information platform of the Ministry of the Interior. Overall real estate price predictions in Taiwan, especially those for three major cities in northern, central, and southern Taiwan were made. The analysis and prediction results showed that the fuzzy time series forecast model applied in this study had error percentages lower than 5% in the annual and seasonal prediction analytic results. Thus, the forecast model has considerably high prediction accuracy.

*Keywords:* Fuzzy time series; Prediction; Real estate price

### Introduction

As a Chinese proverb says, “having land is having wealth.” Real estate

has always been the basic demand of settlement and the optimal hedge option of investment and financial management for people in Taiwan. It is also the leading industry driving national economic development while having enormous influence on the industrial

economic development of the country. For a long time, the price information of the real estate market in Taiwan has been opaque. Some real estate dealers raise prices at will for large amounts of profit, resulting in the unreasonable appreciation of real estate prices in Taiwan. In addition, individual house expenditures are increasing, which severely influences the quality of life for people in Taiwan as well as social residence justice. To moderate the changes in real estate prices for the reasonable development of related market economy, the real estate real price registration system was implemented starting from August 2012 in Taiwan. People in Taiwan are provided with related real estate price information and statistical information, which can be accessed at any time through different types of electronic network interfaces. The open information and data still require further analysis and evaluation to generate useful information for people to grasp the trend and

development of related real estate prices. Thus, the results of such an evaluation would provide the government with assistance in establishing related industrial policies, help people in investment and financial management, and promote effective and reasonable use and planning of income and expenditures for construction-related industries and households.

### Methods and Objectives

This study was conducted using the secondary statistical data of the “sale contract average unit price (not classified by building type)—10,000 NTD/Ping” obtained from the real estate information platform of the Ministry of the Interior. The annual and seasonal trend charts of different time series were established for the whole country as well as for Taipei City, Taichung City, and Kaohsiung City, as shown in Figure 1 and Figure 2.

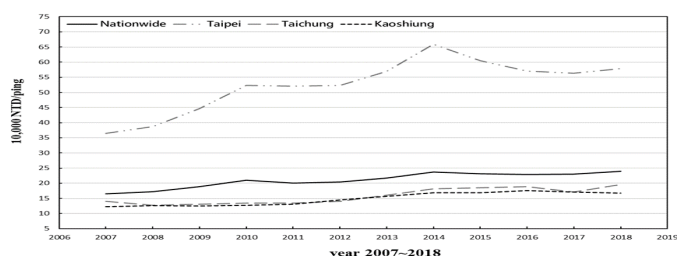


Figure 1. Trend chart of real estate prices in Taiwan from 2007 to 2018.

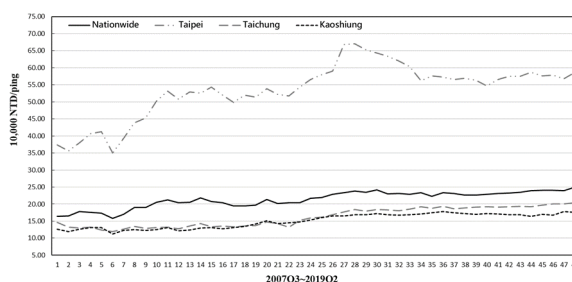


Figure 2. Seasonal real estate price trends in Taiwan.

From the distribution of the curves in the charts, no seasonal or circulating phenomenon was observed, but irregular rising trends were observed. Then, the methods of fuzzy time series forecast models in the literature were referenced for analysis of the trends. First, the annual and seasonal prediction levels of the fuzzy time series method were separately calculated using the aforementioned time series data. Next, the real estate sale contract average unit price (not classified by building type) predicted values for the next year and next season were analyzed for the entire country and the three major cities of Taipei City, Taichung City, and Kaohsiung City. The main study purposes are as follows:

1. To establish a model for predicting future real estate unit price trends in Taiwan by using the fuzzy time series method.
2. To discuss the annual and seasonal prediction levels of fuzzy time series for the “sale contract average unit price (not classified by building type)—10,000 NTD/Ping” for real estate buildings with the time series statistical data of the real price registration in the Taiwan region.
3. To provide real estate personnel (such as government decision makers, investors, construction businesses, and consumers) with a forecast model as a reference.

### **Fuzzy Time Series**

### *Literature Review*

After fuzzy theory was proposed by Zadeh (1965), scholars started to apply this theory to create different research methods, and the related academic research is extensive. Fuzzy theory is based on the fuzzy set with the basic idea of “harmonic and tolerating,” which is different from the “clear standpoints” (crisp sets) of traditional set theory. In the definition of the fuzzy set, for some element  $X$ ,  $\mu(x)$  is used to represent the probability of  $X$  belonging to a particular set. Namely,  $X$  corresponds to a function of  $[0, 1]$ , and the closer the degree is to 1 means a higher probability of that set containing  $X$  element. This value is called the degree of membership, and  $\mu(x)$  is called the membership function. When the value of the membership function,  $\mu(x)$ , can only be 0 or 1, the set is a traditional crisp set. Zadeh (1975) explained that there are numerous situations that are complicated or difficult to define in real life. It is difficult to use traditional quantization methods for the reasonable and clear expression of these situations with qualification standards. Thus, these situations should be addressed from the perspective of linguistic variables.

Song and Chissom (1993b) used a fuzzy time series to dynamically define and study linguistic values, and fuzzy time series is a new approach proposed by Zadeh (1973). The greatest difference between traditional time series and fuzzy time series is that the analytic data type for traditional time series is the real number, and that the type for fuzzy time series is the linguistic variable of the fuzzy set. Thus,

Song and Chissom (1993a) (1994) defined the basic structure of the fuzzy time series model based on Zadeh's fuzzy theory, and different fuzzy time series prediction methods were proposed for time series data. The time series data of the number of newly registered students of the University of Alabama were used as an example, and the method of the establishment of the fuzzy time series forecast model was explained.

Chen (1996) considered the calculation method using max–min compositions to de-fuzzy to be too complicated in the prediction method proposed by Song and Chissom (1993a). A new prediction method for easier calculation was proposed under the fuzzy time series model using the newly registered student number of the University of Alabama as the study data. The empirical results showed the superior outcomes of the study method for predicting the newly registered student number, and that method provided favorable prediction outcomes when the history data were incomplete.

Hwang et al. (1998) adopted the changes in the fuzzy time series data and proposed another forecast model. This analysis also used the newly registered student number from the study by Song and Chissom (1993a) (1994). The trend of the newly registered student number in the previous year was

used for predicting the change in this number in the next year. The changing situation of the previous year was used to find the related fuzzy logical set as the principle predicting changes in the number in the next year. Moreover, the mean absolute percentage error (MAPE) levels of this method were compared with those of different fuzzy time series forecast models. As shown in Table 1 (Hwang et al., 1998), the method developed by Hwang et al. (1998) is superior to the methods developed by Song and Chissom (1993a) (1994) as well as by Chen (1996), and the method has extremely favorable prediction levels. In addition, the proposed method (1998) is more efficient than the ones presented in (Song & Chissom, 1993a, 1994; Sullivan & Woodall, 1994) because the proposed method simplifies the arithmetic operation process.

#### *Fuzzy time series forecast model*

The obtained prediction levels of different fuzzy time series forecast models are compared in Table 1. Although differences exist in MAPE levels, these fuzzy time series forecast models have extremely favorable prediction levels. If MAPE prediction levels proposed by Lewis (1982) are used for judgment, these fuzzy time series forecast models have favorable prediction levels.

Table 1. A comparison of the average forecasting errors of different forecasting methods.

Litera- ture and re-	Song-Chissio m Method (1993a)	Song-Chissio m Method (1994) (under model basis w=4 and us-	Chen's method (1996)	Sulli- van-woodall Markov method (1994)	Hwang, Chen and Lee method (1998) (under

search methods		ing neural net method)			window basis w=4)
Style	Time-invariant	Time-variant	Time-invariant	Time-invariant	Time-variant
Time complexity	$O(kn^2)$	$O(kn^2)$	$O(p)$	$O(cn^2)$	$O(wn)$
Average forecasting errors	3.2%	4.37%	3.22%	2.6%	3.12%

Note:  $k$  denotes the number of fuzzy logical relationships,  $n$  denotes the number of elements in the universe of discourse,  $p$  denotes the number of fuzzy logical relationship groups,  $c$  denotes the number of transitions in the historical data, and  $w$  denotes the window basis.

Reference from: Hwang et al. (1998)

This study adopted Chen's method, which has less complicated analytic calculations and extremely favorable prediction levels (Table 1). The time series statistical data of "sale contract average unit price (not classified by building type)—10,000 NTD/Ping" of Taiwan and the three major cities obtained from the real estate information platform of the Ministry of the Interior in Taiwan was used. The annual and seasonal prediction ability of this study method to predict real estate prices in Taiwan was analyzed.

The prediction analysis steps are as follows:

Step 1: Set  $U$  as the universe of dis-

course of a time series, and  $U = \{u_1, u_2, \dots, u_n\}$ , in which  $u_n$  is a subset of  $U$ . Let  $D_{\max}$  and  $D_{\min}$  be the maximum and minimum values of the times series values in the universe of discourse. Then, range  $R = [D_{\min} - D_1, D_{\max} + D_2]$ , in which  $D_1$  and  $D_2$  are adequate positive numbers such that  $D_{\min} - D_1$  and  $D_{\max} + D_2$  are integers. Next, adequate class numbers are used, so that the class intervals in each class are constant. Then, the ranges were grouped with adequate class numbers, which are  $u_1, u_2, \dots, u_n$ .

The annual real estate average unit prices of Taiwan (2007–2018) are used as an example, as shown in Table 2.

Table 2. Real estate prices of Taiwan (2007–2018) and the fuzzy sets.

Year $t$	Real estate price $Y(t)$ (10,000)	Member ship subset	Median $m_n$	Fuzzy set $A_k$	Fuzzy logical rela-
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	NTD/Ping)	$u_n$			tionship
2007	16.50	$u_1 = [16,$ 17]	16.50	$A_1$	
2008	17.16	$u_2 = [17,$ 18]	17.50	$A_2$	$A_1 \rightarrow A_2$
2009	18.88	$u_3 = [18,$ 19]	18.50	$A_3$	$A_2 \rightarrow A_3$
2010	20.99	$u_5 = [20,$ 21]	20.50	$A_5$	$A_3 \rightarrow A_5$
2011	20.00*	$u_5 = [20,$ 21]	20.50	$A_5$	$A_5 \rightarrow A_5$
2012	20.40	$u_5 = [20,$ 21]	20.50	$A_5$	$A_5 \rightarrow A_5$
2013	21.73	$u_6 = [21,$ 22]	21.50	$A_6$	$A_5 \rightarrow A_6$
2014	23.74	$u_8 = [23,$ 24]	23.50	$A_8$	$A_6 \rightarrow A_8$
2015	23.09	$u_8 = [23,$ 24]	23.50	$A_8$	$A_8 \rightarrow A_8$
2016	22.87	$u_7 = [22,$ 23]	22.50	$A_7$	$A_8 \rightarrow A_7$
2017	22.97	$u_7 = [22,$ 23]	22.50	$A_7$	$A_7 \rightarrow A_7$
2018	23.88	$u_8 = [23,$ 24]	23.50	$A_8$	$A_7 \rightarrow A_8$

\*When this value is equal to the boundary value of the related subset, the overall trend line of the time series is increasing. The larger subset  $u_n$  in the universe of discourse U that the different values of this study analysis belong to.

They are 16.50, 17.16, 18.88, 20.99, 20.00, 20.40, 21.73, 23.74, 23.09, 22.87, 22.97, and 23.88 (10,000 NTD/Ping) in order. The data of 12 years are the universe of discourse U and are used in the analysis of this study.  $D_{\max} = 23.88$  and  $D_{\min} = 16.50$ . According to numerical characteristics, adequate positive numbers  $D_1 = 0.5$  and  $D_2 = 0.12$  are adopted, so that  $R = [16.50 - 0.5, 23.88 + 0.12] = [16, 24]$ . Then, the range value is  $24 - 16 = 8$ . The class interval is 1; thus, the range is divided into eight classes. They are  $u_1 = [16, 17]$ ,  $u_2 = [17, 18]$ ,  $u_3 = [18, 19]$ ,  $u_4 = [19, 20]$ ,  $u_5 = [20, 21]$ ,  $u_6 = [21, 22]$ ,  $u_7 = [22, 23]$ , and  $u_8 = [23, 24]$ .

Step 2: Set  $A_1, A_2, \dots,$  and  $A_k$  as the fuzzy sets with different linguistic variables. Thus, the fuzzy set of  $A_1, A_2, \dots,$  and  $A_k$  is expressed as (1)

$$A_1 = a_{11}/u_1 + a_{12}/u_2 + a_{13}/u_3 \dots + a_{1n}/u_n$$

$$A_2 = a_{21}/u_1 + a_{22}/u_2 + a_{23}/u_3 \dots + a_{2n}/u_n$$

(1)

$$A_k = a_{k1}/u_1 + a_{k2}/u_2 + a_{k3}/u_3 \dots + a_{kn}/u_n$$

$a_{ij}$  denotes the membership function value of  $u_j$  in fuzzy set  $A_i$ . In addition,

$$a_{ij} \in [0,1], 1 \leq i \leq k, 1 \leq j \leq n, \text{ and } k = n.$$

The series values are rewritten with fuzzy sets according to their in-

tervals, so that the data becomes a time series of the fuzzy linguistic type. As shown in Table 2, according to the subset  $u_n$  that the annual actual statistical values  $Y(t)$  ( $t = 2007, 2008, 2009, \dots, 2018$ ) belongs to, the corresponding fuzzy set  $A_k$  of the linguistic variables are filled in the fuzzy set fields in the table. For example,  $Y(2012) = 20.40$  belongs to subset  $u_5$ , and its corresponding linguistic variable fuzzy set is  $A_5$ . The basic prediction principle of this study is that the fuzzy set of the previous time series value  $Y(t - 1)$  influences the current time series value  $Y(t)$ . Consequently, the fuzzy logical relationship of every year is set as  $Y(t - 1) \rightarrow Y(t)$  to obtain

the fuzzy logical relationship  $A_i \rightarrow A_k$ . This indicates that the  $t - 1$  series value belongs to fuzzy set  $A_i$ , and the  $t$  series value belongs to fuzzy set  $A_k$ .

Step 3: The fuzzy logical relationships are established. The logical relationship  $A_i \rightarrow A_k$  is established according to the fuzzy sets from the different fuzzy logical relationships obtained in Step 2 (as shown in Table 2). The groups correspond from top to bottom to the fuzzy logical relationship groups shown in Table 3, with  $A_i$  as the grouping standard.

Table 3. Fuzzy logical relationship groups.

Group1:	$A_1 \rightarrow A_2$	
Group2:	$A_2 \rightarrow A_3$	
Group3:	$A_3 \rightarrow A_5$	
Group4:	$A_4 \rightarrow$	
Group5:	$A_5 \rightarrow A_5$	$A_5 \rightarrow A_6$
Group6:	$A_6 \rightarrow A_8$	
Group7:	$A_7 \rightarrow A_7$	$A_7 \rightarrow A_8$
Group8:	$A_8 \rightarrow A_8$	$A_8 \rightarrow A_7$

The group numbers are the groups numbers set from the calculations in Step 1. The cases calculated in this study are divided into eight groups. If there are no related fuzzy sets after the analysis of fuzzy membership relationships, then that group is a blank space or is labeled as  $A_n \rightarrow$  in Table 3. For example, if the calculated case is in the fourth fuzzy set  $A_4$ , then Group 4 of the fuzzy logical relationships in Table 3 is a blank space or is labeled as  $A_4 \rightarrow$ .

Step 4: The analysis and calculation of predicted value  $F(t)$  ( $t = 2008, 2009,$

$2010, \dots, 2018$ ).

There are three conditions for analysis and calculation, which are as follows:

- (1) If the fuzzy set of the  $t - 1$ -th period is  $A_i$ , and only one fuzzy logical relationship  $A_i \rightarrow A_k$  exists, then the median  $m_n$  of the subset  $u_n$ , which has membership relations with  $A_k$ , is taken as the predicted value of the  $t$ -th period. For example, Group 3 is in the fuzzy logical relationship group of  $A_3 \rightarrow A_5$  in Table 3. Consequently, if the real estate price

$Y(t - 1)$  is in  $u_3$  (membership fuzzy set is  $A_3$ ), then the prediction real estate price of the next year  $F(t)$  is the median  $m_5 = 20.50$  of the subset  $u_5$  that  $A_5$  belongs to.

(2) If the fuzzy set of the  $t - 1$ -th period is  $A_i$ , but there are two or more fuzzy logical relationships  $A_i \rightarrow A_{k1}$ ,  $A_i \rightarrow A_{k2}$ , ...,  $A_i \rightarrow A_{kp}$ , then the  $t$ -th period predicted value is the arithmetic mean of medians  $m_{n1}$ ,  $m_{n2}$ , ..., and  $m_{np}$  of the subsets  $u_{n1}$ ,  $u_{n2}$ , ..., and  $u_{np}$  that have membership relationships with  $A_{k1}$ ,  $A_{k2}$ , ..., and  $A_{kp}$ , respectively. For example, is the fuzzy logical relationship group of Group 5 is one-to-many, then  $A_5 \rightarrow A_5$  and  $A_5 \rightarrow A_6$ . Consequently, when the real estate price  $Y(t - 1)$  is in  $u_5$  (membership fuzzy set is  $A_5$ ), the prediction real estate price  $F(t)$  for the next year is the arithmetic mean of the medians  $m_5$  and  $m_6$  of the subsets  $u_5$  and  $u_6$  that  $A_5$  and  $A_6$  belong to. Thus,  $F(t) = \frac{1}{2} (m_5 + m_6) = \frac{1}{2} (20.5 + 21.5) = 21$ .

(3) If there is no fuzzy logical relationship for the  $t - 1$ -th period fuzzy set  $A_i$ , then the median  $m_n$  of the subset  $u_n$  with that  $A_i$  has membership relationships is taken as the predicted value of the  $t$ -th period. For example,

Group 4 in Table 3 has no fuzzy logical relationship group; thus,  $A_4 \rightarrow \emptyset$ . Consequently, if the real estate price  $Y(t - 1)$  is in  $u_4$  (membership fuzzy set is  $A_4$ ), then the prediction real estate price for the next year  $F(t)$  is the median  $m_4 = 19.50$  of the subset  $u_4$  that  $A_4$  belongs to.

The judgment for the prediction levels used in this study is calculated with MAPE, and the formula is as follows (2):

$$MAPE = \frac{1}{n-1} \left( \sum_{i=2}^n \left| \frac{Y(t)_i - F(t)_i}{Y(t)_i} \right| \right) \times 100 \quad (2)$$

## Results

### *Fuzzy time series prediction of the annual real estate average unit price*

This study used the annual statistical time series data (the statistical numbers of each season in every year from 2007 to 2018 are added and averaged to be the annual values) of the “sale contract average unit price (not classified by building type)—10,000 NTD/Ping” of Taiwan and the three major cities in the north (Taipei City), middle (Taichung City), and south (Kaohsiung) Taiwan from the real estate information platform of the Ministry of Interior (2019). Table 4 shows the results of the fuzzy time series forecast model by Chen (1996) applied for calculation and analysis.

Table 4. Prediction results of the annual sale contract average unit price (not classified by building type) of buildings in Taiwan.

Year	Nationwide	Taipei	Taichung	Kaoshiung
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(t)	Original value Y(t)	Forecast value F(t)	Original value Y(t)	Forecast value F(t)	Original value Y(t)	Forecast value F(t)	Original value Y(t)	Forecast value F(t)
2007	16.50		36.46		13.96		12.27	
2008	17.16	17.50	38.72	41.63	12.67	13.50	12.54	12.75
2009	18.88	18.50	44.62	41.63	13.03	13.50	12.42	12.75
2010	20.99	20.50	52.31	52.88	13.47	13.50	12.68	12.75
2011	20.00	21.00	52.03	54.75	13.46	13.50	13.11	12.75
2012	20.40	21.00	52.27	54.75	14.01	13.50	14.51	14.63
2013	21.73	21.00	56.97	54.75	16.07	13.50	15.66	15.38
2014	23.74	23.50	65.86	60.38	18.11	18.50	16.87	16.88
2015	23.09	23.00	60.49	60.38	18.51	19.00	16.90	17.25
2016	22.87	23.00	57.08	56.63	18.90	19.00	17.50	17.25
2017	22.97	23.00	56.29	60.38	19.18	19.00	17.07	16.88
2018	23.88	23.00	57.89	60.38	19.58	19.50	16.79	17.25
2019	Unknown value	23.00	Unknown value	60.38	Unknown value	19.50	Unknown value	17.25
w = 8	2.13%		4.55%		3.36%		1.60%	
	MAPE		MAPE		MAPE		MAPE	

Notes. In the table,  $w$  denotes the group number of membership subset  $u_n$  used in the execution of fuzzy time series. For example, when  $w = 8$ , there are eight groups; MAPE is the mean absolute error percentages of the predicted values and original values of 2008–2018.

The prediction error percentages were 2.13% for Taiwan, 4.55% for Taipei City, 3.36% for Taichung City,

and 1.60% for Kaohsiung City. As shown in Table 5, prediction levels are extremely favorable.

Table 5. Mean absolute percentage error (MAPE) prediction levels (1982)

MAPE(%)	<10	10-20	20-50	>50
Prediction level	Favorable prediction	Fine prediction	Reasonable prediction	Incorrect prediction

In addition, the unknown “sale contract average unit price (not classified by building type)—10,000 NTD/Ping” of the next year was predicted. For example, the unit prices were 230,000 NTD/Ping for Taiwan, 603,800 NTD/Ping for Taipei City, 195,000 NTD/Ping for Taichung City, and 172,500 NTD/Ping for Kaohsiung City. The trend lines of the real values and predicted values of real estate prices of each region are shown in Figure 3.

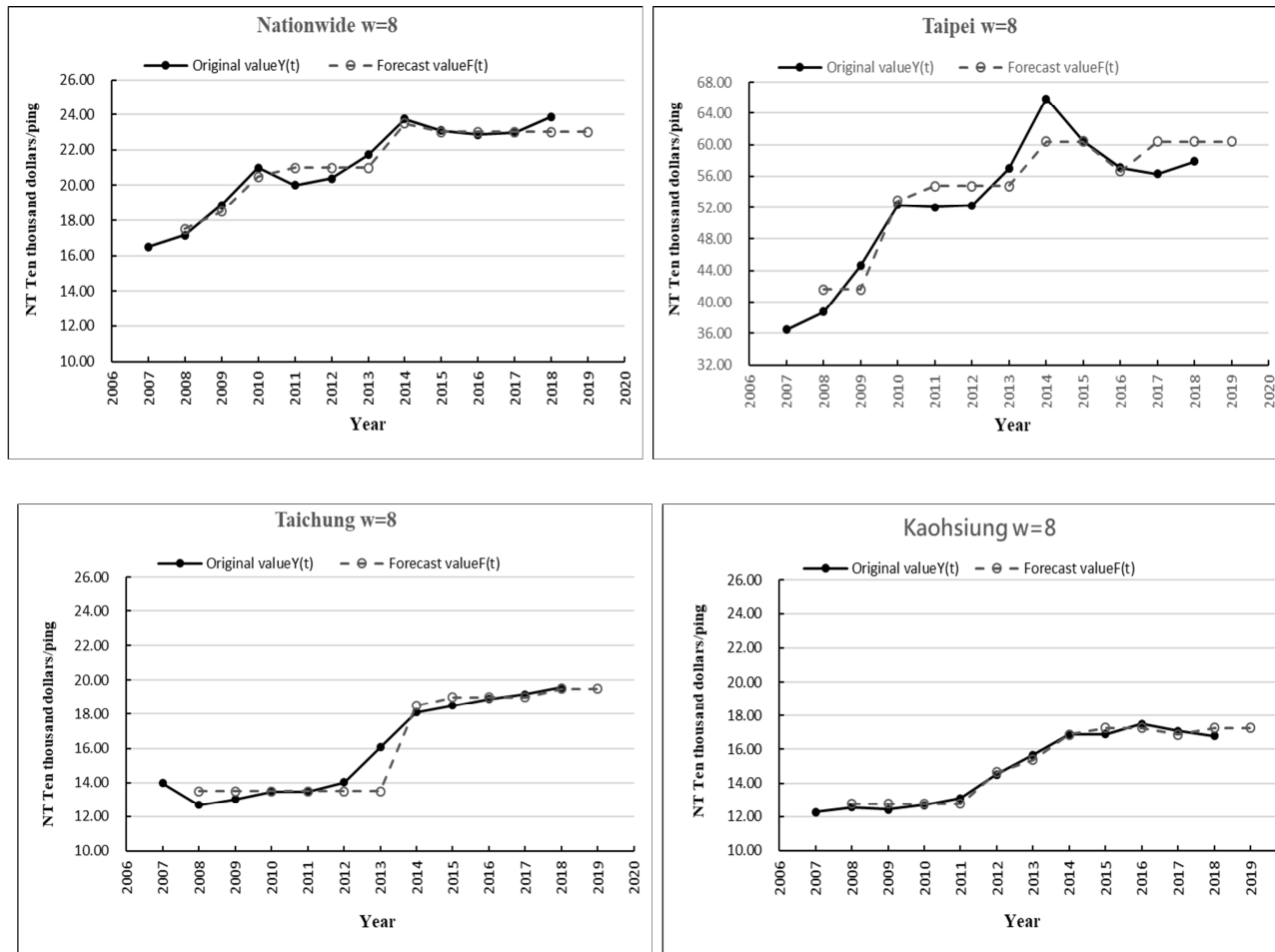


Figure 3. Results of the annual real estate price prediction for Taiwan and the three major cities.

According to the analytical calculation characteristics of the fuzzy time series forecast model proposed by Chen (1996), the larger the number of the fuzzy membership subset groups are, the lower the class intervals of the related membership are. With the fuzzy universe of discourse classification of the history time series data, the predicted values of the next year should be closer to the real values. Thus, the larger the number of fuzzy membership subset groups are, the lower the related error percentages are. In this

study, the numbers of fuzzy membership subset groups can be set to reasonable numbers according to the numerical values of the range R of the history time series data. The set group numbers should be easy to classify and should have favorable prediction levels. Consequently, in this paper, the comparison of prediction levels is further conducted with different fuzzy membership subset group numbers ( $w = 5, 8, \text{ and } 10$ ). The results are shown in Figure 4.

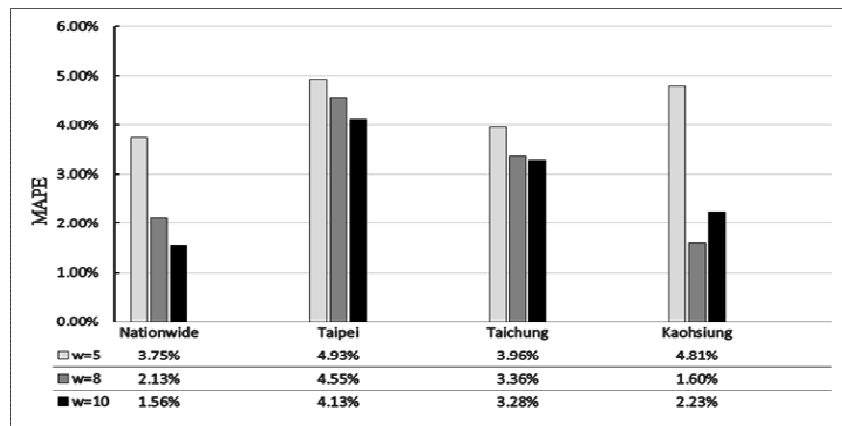


Figure 4. Annual prediction accuracy comparison of different regions and different membership subset group numbers. MAPE: mean absolute percentage error.

In addition to the prediction level of  $w = 8$  being superior to the prediction level of  $w = 10$  for Kaohsiung City, the test results show that the larger the fuzzy membership subset group numbers are, the more favorable the predictive power is, and the smaller the error percentages for the predicted and real values are. Overall, for the prediction levels obtained from the fuzzy membership subset group numbers applied in this study ( $w = 5, 8, \text{ and } 10$ ), the error percentages are all smaller than 5%, reaching considerably favorable prediction levels.

#### *Fuzzy time series prediction of seasonal real estate unit price*

In this subsection, the “sale contract average unit price (not classified by building type)—10,000 NTD/Ping” analytic time series data in every season in the last 3 years (2016Q1–2019Q2) of the entire country and the three major cities in the northern (Taipei City), middle (Taichung City), and southern (Kaohsiung City) Taiwan is used, which was obtained from the real estate information platform of the Ministry of the Interior [11]. The fuzzy time series forecast model by Chen [7] was applied for calculation and analy-

sis using the membership subset group number  $w = 10$ , and the results are shown in Table 6.

Table 6. Prediction results of the annual sale contract average unit price (not classified by building type) of buildings in Taiwan.

Season ( $t_q$ )	Nationwide		Taipei		Taichung		Kaoshiung	
	Original value $Y(t_q)$	Forecast value $F(t_q)$	Original value $Y(t_q)$	Forecast value $F(t_q)$	Original value $Y(t_q)$	Forecast value $F(t_q)$	Original value $Y(t_q)$	Forecast value $F(t_q)$
2016Q1	22.34		57.59		18.76		17.47	
2016Q2	23.36	23.40	57.27	57.50	19.35	19.05	17.86	17.50
2016Q3	23.17	23.40	56.60	57.75	18.61	19.20	17.43	17.50
2016Q4	22.60	23.00	56.85	57.38	18.89	19.05	17.25	17.50
2017Q1	22.64	22.80	56.31	57.38	19.15	19.05	16.94	17.00
2017Q2	22.89	22.80	54.71	54.75	19.27	19.35	17.27	17.15
2017Q3	23.12	23.00	56.60	56.75	19.07	19.20	17.13	17.00
2017Q4	23.24	23.00	57.54	57.38	19.23	19.35	16.93	16.90
2018Q1	23.51	23.40	57.48	57.50	19.32	19.20	16.92	17.15
2018Q2	23.94	23.40	58.64	57.75	19.26	19.20	16.45	17.15
2018Q3	24.05	24.60	57.64	57.75	19.68	19.20	16.98	16.90
2018Q4	24.03	24.00	57.81	57.50	20.05	19.95	16.80	17.15
2019Q1	23.96	24.00	56.81	57.50	20.07	20.25	17.84	17.15
2019Q2	25.01	24.60	58.71	57.38	20.40	20.25	17.61	17.70
2019Q3	Unknown value	25.00	Unknown value	57.75	Unknown value	20.55	Unknown value	17.70
$w = 10$	0.96%		0.89%		1.02%		1.42%	
	MAPE		MAPE		MAPE		MAPE	

Notes: In the table,  $w$  denotes the group number of the membership subset  $u_n$  used in the execution of fuzzy time series. For example, when  $w = 10$ , there are 10 groups; MAPE is the mean absolute percentage error of the predicted values and original values of 2016Q2–2019Q2.

The prediction error percentages were 0.96% for Taiwan, 0.89% for Taipei City, 1.02% for Taichung City, and 1.42% for Kaohsiung City. According to Table 5, they all have extremely favorable prediction levels. In addition, the unknown “sale contract average unit price (not classified by building type)—10,000 NTD/Ping” real estate prices in the next season (2019Q3) were predicted. It was found to be 250,000 NTD/Ping for Taiwan, 577,500 NTD/Ping for Taipei City, 205,500 NTD/Ping for Taichung City,

and 177,000 NTD/Ping for Kaohsiung City. The trend lines of the real values and predicted values of real estate prices in each region are shown in Figure 5.

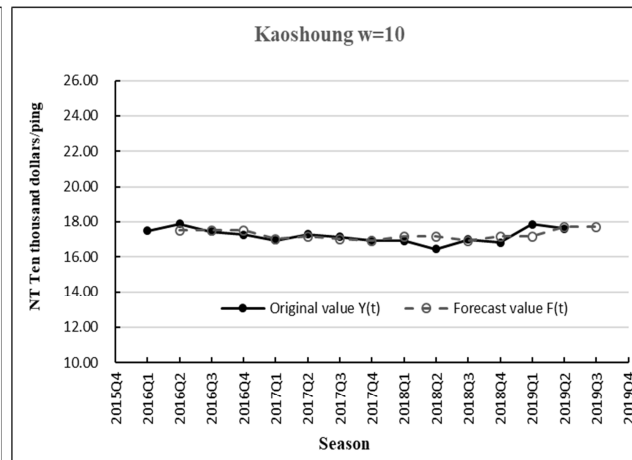
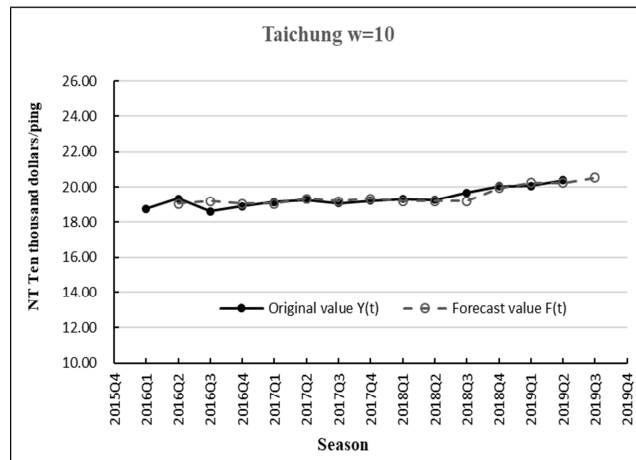
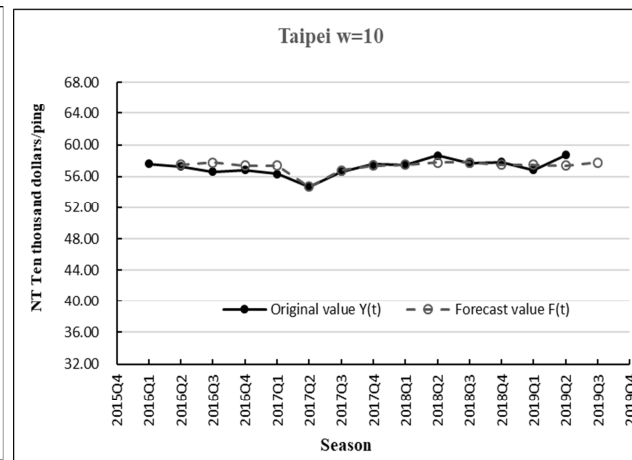
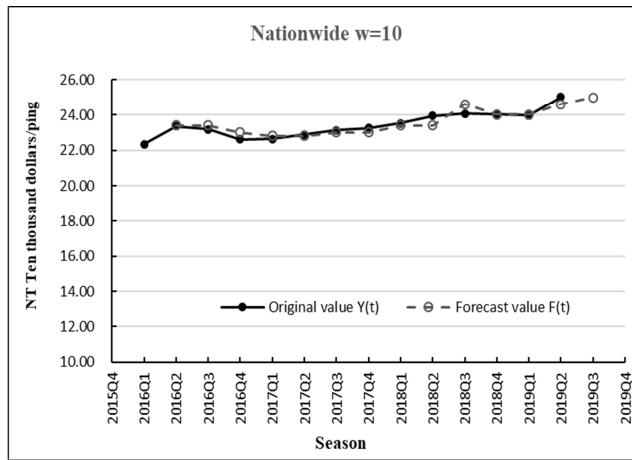


Figure 5. Results of the seasonal real estate price prediction for Taiwan and the three major cities.

According to the analytic calculation characteristics of the fuzzy time series forecast model by Chen (1996), the comparison of the prediction levels of seasonal time series data was conducted with different fuzzy membership subset group numbers ( $w = 5, 8, \text{ and } 10$ ). The results are shown in Figure 6.

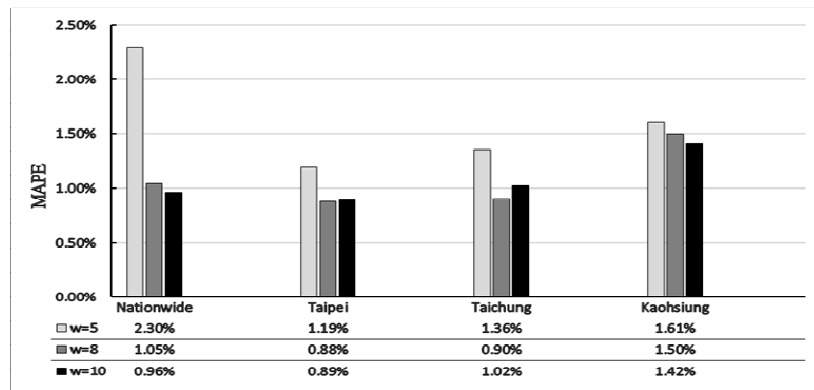


Figure 6. Seasonal prediction accuracy comparison of different regions and different membership subset group numbers. MAPE: mean absolute percentage error.

In addition to the prediction levels of  $w = 8$  being superior to the prediction levels of  $w = 10$  in Taipei City and Taichung City, the test results indicate that the larger the fuzzy membership subset group numbers are, the more favorable the predictive power is, and the smaller the error percentages between the predicted and real values are. However, for the fuzzy membership subset group numbers of  $w = 8$  and  $w = 10$ , the analyzed prediction levels are extremely close, with not much difference. Overall, for the prediction levels obtained with the applied fuzzy membership subset group numbers ( $w = 5, 8, \text{ and } 10$ ), the error percentages are all smaller than 3%. Thus, they all belong to extremely favorable prediction levels.

### Conclusion

In this study, the fuzzy time series forecast model of Chen (1996) was used to analyze and calculate the re-

sults. Different fuzzy membership subset group numbers were used to compare the prediction levels with the same history time series data. In both annual and seasonal analyses, the same results were obtained for the differences in predicted and real values. For most of the fuzzy membership subsets, the more the group numbers are, the more favorable the prediction abilities are. However, in three occasions, the prediction level with  $w = 8$  is superior to that with  $w = 10$ . This indicates that some exceptions exist when using the fuzzy time series prediction model proposed by Chen (1996) for different time series data. Thus, different group numbers should be analyzed to obtain the optimal prediction ability before executing the prediction operation of the next unknown number. Then, it is best to execute related prediction operations with the optimal group numbers. In addition, the trend lines of the time series data of Figure 1 and Figure 2 and the results of this study show that the larger the history time series

data changes are, the larger the error percentages obtained from the comparison are. However, the overall prediction levels obtained from the applied fuzzy membership subset group numbers ( $w = 5, 8, \text{ and } 10$ ) of the fuzzy time series forecast model by Chen (1996) all have error percentages of lower than 5%. Thus, they all belong to extremely favorable prediction levels.

The conclusions after the analysis and calculation according to the purposes of this study are as follows:

1. The fuzzy time series forecast model of Chen (1996) had the characteristics of simple calculation and analysis. This model is an extremely favorable forecast model that used small amounts of history time series change data (approximately 12–14 pieces) and fuzzy membership subset group numbers ( $w = 5, 8, \text{ and } 10$ ) for predicting the future real estate unit price in Taiwan.
2. The aforementioned fuzzy time series forecast model was applied to the time series statistics of the actual registered prices in Taiwan. For the prediction levels obtained from annual and seasonal analyses, the error percentages are all smaller than 5%. They have little difference with the prediction levels of the different fuzzy time series forecast models shown in Table 1. The error levels obtained from seasonal time series data analysis are superior to those of the other different fuzzy time series forecast models in Table 1.
3. The fuzzy time series forecast

model applied in this study requires small amounts of history time series data and has simple analysis and calculation characteristics. Thus, this forecast model can be provided to real estate personnel (such as government policy makers, investors, construction businesses, and consumers) as a reference.

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